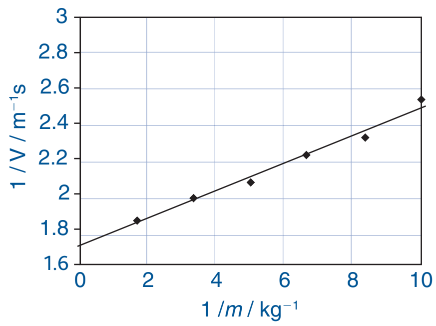
Answers to examination-style questions

Answers	Marks	Examiner's tips
1 (a) There is a much greater gap between the mean values of x_B for the 0.100 and 0.200 kg masses than there is between the 0.200 and 0.300 kg masses. Measurements for two more values of *m* between 0.100 and 0.200 kg would bridge the gap. Also, for equal increases of *m,* the difference between successive mean values becomes less and less so at least one further measurement significantly above 0.300 kg would extend the range of mean values significantly	**4**	
(b) $<x_B> = 86.4$ mm, $\theta = 12.19°$	**2**	It is the normal practice in tables of experimental results to maintain a consistent number of decimal places down a column. In this case $<x_B>$ is given to 1 and the angle to 2 decimal places. The last decimal place cannot be justified on grounds of accuracy.
(c) (i) Loss of kinetic energy of B after collision = gain of potential energy to maximum height *h* $\frac{1}{2}MV^2 = Mgh$ rearranging gives $V = \sqrt{2gh}$	**2**	
(ii) $h = 12.96$ mm, $V = 0.504$ m s^{-1}	**2**	Since $g = 9.81$ m s^{-2} and *V* is in m s^{-1} the value of *h* must be converted from mm to m before the calculation.
(d) (i) 	**3**	3 marks for: • suitable scales, • correct labels, • points plotted correctly, • best fit line drawn.
(ii) k = y-intercept = 1.79 m^{-1} s, kM = gradient, gradient of graph $= \dfrac{(2.75 - 1.75)}{10}$ $= 0.10$ m^{-1} s kg^{-1} $M = \dfrac{gradient}{k} = \dfrac{0.10}{1.79}$ $= 5.6 \times 10^{-2}$ kg	**4**	

Answers	Marks	Examiner's tips
(e) (i) The graph is a straight line with a positive gradient and a positive intercept. The theoretical equation is the equation for a straight line with a positive gradient kM and a positive y-intercept k. The graph is based on experimental measurements and it agrees with the theoretical equation so the theory behind the equation is valid.	**3**	
(ii) The values of k and M are reliable because the graph is clearly a straight line and the theoretical equation supports it.		

m^{-1}/kg^{-1}	V^{-1} / m^{-1} s
10.0	2.85
8.33	2.58
6.67	2.44
5.00	2.26
3.33	2.15
1.67	1.98

Answer to Extension Questions

Answers	Marks	Examiner's tips
(f) Conservation of momentum gives $mu = mv + MV$	**5**	**Note** the "theoretical equation" to be derived is in Q1**(d)**.

Combining this equation with $(V - v) = e\,u$ to eliminate v gives
$mu = m(V - e\,u) + MV$

Rearranging this equation gives
$mu + m\,e\,u = mV + MV$

Dividing both sides by mV gives

$$\frac{u(1+e)}{V} = 1 + \frac{M}{m}$$

Dividing each term by $u\,(1 + e)$ gives

$$\frac{1}{V} = \frac{1}{u(1+e)} + \frac{M}{u(1+e)}\frac{1}{m}$$

which is the same as the theoretical

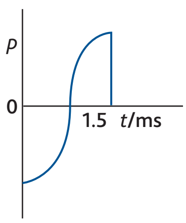
equation with $k = \dfrac{1}{u(1+e)}$

Answers to examination-style questions

Answers	Marks	Examiner's tips
2 (a) In an inelastic collision, **kinetic** energy is not conserved	1	Momentum is conserved in all collisions, but kinetic energy is conserved only when a collision is perfectly elastic. Don't fall into the trap of saying that **energy** is not conserved: it must be **kinetic energy** that you mention.
(b) (i) Momentum $p = mv$ gives $p = 0.12 \times 18 = 2.16$ N s (or kg m s^{-1})	1	Simple substitution of the given values provides the answers to these parts easily, but do remember that momentum is a **vector** and that its direction matters. Hitting the ball reverses its direction of travel.
(ii) $p = 0.12 \times (-15)$ $= -1.8$ N s (or kg m s^{-1})	1	
(iii) Change in momentum $= 2.16 - (-1.8)$ $= 3.96$ N s (or kg m s^{-1})	1	
(iv) Average force $F = \frac{\Delta p}{\Delta t} = \frac{3.96}{0.14}$	1	"Force = rate of change of momentum" is the fundamental consequence of Newton's second law of motion.
$= 28$ N	1	
(v) Kinetic energy lost $E_k = \frac{1}{2} \times 0.12 \times (18^2 - 15^2) = 5.9$ J	1	This confirms that the collision with the bat was inelastic. If the collision had been elastic, the speed of the ball would have been 18 m s^{-1} after impact.
3 (a) Impulse $= F\,\Delta t =$ area under graph	1	The area to be found is that of a simple triangle of height 1.8 N and base 0.15 s. The answer could be expressed in kg m s^{-1} instead of Ns. This same impulse is given to each of the carts.
$= \frac{1}{2} \times 1.8 \times 0.15 = 0.135$ N s	1	
(b) Impulse = change of momentum $\therefore 0.135 = m_A \times 0.60$	1	The question states that cart A is moving at 0.60 m s^{-1} when the spring drops away. The impulse is equal to the momentum gained by each cart.
from which $m_A = 0.225$ kg or 0.22 kg or 0.23 kg	1	
(c) The final total momentum of the system is zero.	1	From the impulse, each cart receives momentum of the same magnitude. But momentum is a vector and the carts move in opposite directions. Therefore the **total momentum** of the **system** is 0.135 + (–0.135) = 0.
4 (a) (i) $p = mv = 6.2 \times 10^4 \times 0.35$	1	An easy two marks for showing that you know what momentum is, but a correct unit is essential for full credit.
$= 2.17 \times 10^4$ N s (or kg m s^{-1}) or 2.2×10^4 N s	1	
(ii) Initial momentum of engine = combined momentum after coupling	1	Momentum is conserved when the engine couples to the carriage, because the only forces acting on the system are internal forces. The mass of the combined system is $(6.2 + 4.0) \times 10^4 = 10.2 \times 10^4$ kg.
$\therefore 2.17 \times 10^4 = 10.2 \times 10^4\, v$	1	
gives $v = 0.213$ m s^{-1} or 0.21 m s^{-1}	1	

Answers	Marks	Examiner's tips
(b) Kinetic energy of train $= \frac{1}{2} \times 10.2 \times 10^4 \times 0.15^2 = 1150$ J Elastic energy stored in spring $= \frac{1}{2} F e$ $= \frac{1}{2} \times (k\,e) \times e = \frac{1}{2} k\,e^2$ $\frac{1}{2} \times 320 \times 10^3\, e^2 = 1150$ gives compression $e = 8.47 \times 10^{-2}$ m or 8.5×10^{-2} m	1 1 1 1	All of the kinetic energy of the train is stored in the spring when fully compressed, because the train has then been stopped. Note that the speed of impact is given as 0.15 m s^{-1} in the question; the train has slowed after coupling together. Elastic energy is covered in Unit 2 of *AS Physics A*. The *stiffness* of a spring is sometimes called the *spring constant*.
5 (a) *Two quantities that are conserved*: • momentum • **kinetic** energy	2	One mark for each. Momentum is conserved in all collisions. An elastic collision (sometimes called a perfectly elastic collision) is special, because there is no loss of kinetic energy.
(b) (i) Magnitude of velocity is 450 m s^{-1} The direction is away from the wall at 90° to it (or in the opposite direction to the initial velocity).	1 1	Since the collision is elastic, there is no loss of kinetic energy. The speed of the molecule must therefore be unchanged, but it has rebounded in the opposite direction.
(ii) Initial momentum $= 8.0 \times 10^{-26} \times 450$ N s (or kg m s^{-1}) final momentum $= -8.0 \times 10^{-26} \times 450$ N s (or kg m s^{-1}) change in momentum $= 7.2 \times 10^{-23}$ N s (or kg m s^{-1})	1 1	The momentum of the molecule is reversed in the collision, so its change in momentum is twice as large as it would be if the molecule were simply brought to rest.
(c) *Relevant points*: • Force is exerted **on** the molecule **by** the wall • Molecule experiences a change in its momentum • Molecule must exert a force **on** the wall which is equal and opposite to the force produced by the wall on the molecule, by Newton's third law of motion	4	The change of momentum of the molecules is caused by the wall when it exerts a force on them. The question asks for an explanation of why there is a force on the wall, and requires a reference to the appropriate Newtonian law.
6 (a) ${}^{210}_{84}\mathrm{Po} \rightarrow {}^{4}_{2}\alpha + {}^{206}_{82}\mathrm{Pb}$	2	Part **(a)** revises the α decay equation, covered in Unit 1 of *AS Physics A*. One mark for both nucleon numbers correct (4, 206) and one mark for both proton numbers correct (2, 82).

Answers to examination-style questions

<table>
<tr><th>Answers</th><th>Marks</th><th>Examiner's tips</th></tr>
<tr><td>(b) (i) Mass of $\alpha = 4.0 \times 1.66 \times 10^{-27}$
$= 6.64 \times 10^{-27}$ kg
E_K of $\alpha = \frac{1}{2} \times 6.64 \times 10^{-27} \times (1.6 \times 10^7)^2$
$= 8.50 \times 10^{-13}$ J
$= \dfrac{8.50 \times 10^{-13}}{1.60 \times 10^{-13}} = 5.3$ MeV</td><td>1
1
1</td><td>Remember that the mass must be in kg when substituting in $\frac{1}{2} m v^2$.
$1u = 1.66 \times 10^{-27}$ kg is given in the Data Booklet.
Also, 1 eV $= 1.60 \times 10^{-19}$ J, so 1 MeV $= 1.6 \times 10^{-13}$ J.</td></tr>
<tr><td>(ii) Momentum is conserved in the explosion as the α is emitted, hence
$m_{Pb} v_{Pb} = m_\alpha v_\alpha$
$206\, v_{Pb} = 4 \times 1.6 \times 10^7$
gives $v_{Pb} = 3.1 \times 10^5$ m s^{-1}</td><td>1
1
1</td><td>The only forces acting during an explosion are internal to the system, so momentum is conserved.
$m_{Pb} = 206$ u and $m_\alpha = 4$ u.
There is no need to convert these masses into kg, because the same conversion would apply to both sides of the equation.</td></tr>
<tr><td>7 (a) The shaded area represents impulse (or change in momentum).</td><td>1</td><td>"Momentum" (without "change in") would not be an acceptable answer.</td></tr>
<tr><td>(b) Initial momentum of ball
= half of the shaded area
$= \frac{1}{2} \times (\frac{1}{2} \times 1.6 \times 10^{-3} \times 1.7)$
$= 6.8 \times 10^{-4}$ N s (or kg m s^{-1})</td><td>1
1
1</td><td>You have to recognise that the ball will stop (and lose all its initial momentum) at the point where the force is a maximum. The area required is one half that which is shaded, rather than all of it. The area could be found by counting squares, but that would be tedious. The shape is clearly a triangle, and calculation leads to a quick result.</td></tr>
<tr><td>(c) Graph to show:
• Axes labelled and any line showing a reduction in negative momentum and an increase in positive momentum
• Correct shape of curve

</td><td>2</td><td>The force acting on the ball increases as it is brought to rest, so the (negative) acceleration increases. The momentum-time graph will be proportional to a velocity-time graph, where an increasing acceleration is shown by an increasing gradient. The process is reversed as the ball starts to move upwards, that is, the upwards acceleration decreases with time. Credit would be given for a graph which started with positive momentum and ended with negative momentum.</td></tr>
<tr><td>8 (a) The total momentum of a system of objects remains constant
provided that no external resultant force acts on the system</td><td>1
1</td><td>Momentum is therefore conserved in collisions and in explosions, irrespective of whether there is any change in the kinetic energy of the system. Note that it is important to include the condition…no external force….when stating this principle.</td></tr>
</table>

Answers	Marks	Examiner's tips
(b) (i) **In 1 second**, volume of water entering or leaving nozzle $= \frac{\text{mass}}{\text{density}} = \frac{0.31}{1000} = 3.1 \times 10^{-4}\ \text{m}^3$	**1**	It is useful to consider a time of 1 second in this kind of calculation. You can then imagine the cylinder of water that emerges from the nozzle in 1s; its length will be numerically equal to the speed of the water.
speed of water $= \frac{\text{volume}}{\text{c.s.area}} = \frac{3.1 \times 10^{-4}}{7.3 \times 10^{-5}} = 4.25\ \text{m s}^{-1}$	**1**	
(ii) Change in velocity of water $= 4.25 - 0.68 = 3.57\ \text{m s}^{-1}$	**1**	This calculation is based on "force = rate of change of momentum", which is the change in momentum in 1 s. Strictly, the answer is the force acting on the water owing to its change in momentum, but an equal and opposite force must act on the hose.
change in momentum in 1 s $= 0.31 \times 3.57 = 1.11\ \text{N s}$	**1**	
$F = \frac{\Delta(mv)}{\Delta t}$ gives $F = \frac{1.11}{1.0} = 1.11\ \text{N}$ or 1.1 N	**1**	
(iii) The water jet produces a force on the wall, whilst a force of equal magnitude acts on the hose.	**1**	The water jet acts like an imaginary rod, connecting the nozzle of the hose to the wall. Such a rod would produce equal and opposite pushes at its two ends, so there would be no overall effect on the rotation of the Earth.
The force on the hose is transmitted to the Earth through its support, and this force is in the opposite direction to the force on the wall	**1**	

Answers to examination-style questions

Answers	Marks	Examiner's tips
1 (a) Each spring holds its brake pad retainer on the shaft at low speed.	1	
If the rotation speed is increased, the brake pad retainer moves away from the shaft and compresses the spring, which acts against the outward movement of the retainer.	1	
If the rotation speed is fast enough, the spring is unable to prevent the brake pad coming into contact with the collar.	1	
Friction between the brake pad and the collar prevents the shaft rotating any faster.	1	max 3 marks for **(a)**
(b) For no braking, the centripetal force < 250 N.	1	When the centripetal force exceeds 250 N the spring extends outwards and, as the gap is small, braking occurs.
$\therefore m\omega_0^2 r = 250$ N at the maximum angular speed ω $0.30\ \omega_0^2 \times 0.060 = 250$	1	
$\omega_0^2 = \dfrac{250}{0.30} \times 0.060 = 1.39 \times 10^4$ rad^2 s^{-2}	1	
$\omega_0 = 118$ rads^{-1} Maximum frequency of rotation $= \dfrac{\omega_0}{2\pi}$ $= 19$ Hz	1	
(c) If the springs became weaker, the tension in the springs at which the brake pads touched the collar would be less …	1	
… so braking would occur at a lower rotation frequency.	1	
The lifeboat would descend at a lower speed, or more friction occurs.	1	max 2 marks for **(c)**
2 (a) *Relevant points include:* • Speed is the magnitude of velocity (or speed is a scalar but acceleration [*or velocity*] is a vector). • In circular motion at constant speed the direction of motion changes continuously. • Therefore the velocity is changing. • Acceleration is the rate of change of velocity.	3	Alternatively, this can be argued as follows: • speed is the magnitude of velocity (or speed is a scalar but acceleration [*or velocity*] is a vector) • force (or acceleration) acts towards the centre of the circle • force (or acceleration) is always perpendicular to the velocity (or has no component in the direction of the velocity) • so the force changes the direction of the velocity but not its magnitude

Answers to examination-style questions

Answers	Marks	Examiner's tips
(b) Angular speed $\omega = 2\pi f = 2\pi \times \frac{78}{60}$ $= 8.17$ rad s^{-1}	**1**	The frequency f is the number of revolutions **per second**, and the angular speed is found by multiplying this by 2π. It is possible (although more tedious) to calculate the answer using $F = \frac{mv^2}{r}$, provided you remember that $\frac{2\pi r}{T}$, and that the period T is $\frac{60}{78}$ s.
Maximum frictional force F = centripetal force $\therefore F = m\,\omega^2\,r$ gives $0.50 = 0.10 \times 8.17^2 \times r$	**1**	
from which maximum distance $r = 7.5 \times 10^{-2}$ m	**1**	If the distance from the axis were greater than 7.5×10^{-2} m, the centripetal force required to hold the mass on the table would increase; the maximum frictional force of 0.50 N would no longer prevent the mass from being thrown off.
3 (a) Use of $\frac{2\pi r}{T}$, where $T = \frac{60}{45}$ s	**1**	*Alternatively*: Angular speed $\omega = 2\pi f = 2\pi \times \frac{45}{60}$ $= 4.71$ rad s^{-1}
gives $v = \frac{2\pi r}{T} = \frac{2\pi \times 0.125}{1.33}$	**1**	
$= 0.59$ m s^{-1}	**1**	Linear speed $v = \omega r = 4.71 \times 0.125$ $= 0.59$ m s^{-1}
(b) (i) Radial arrow drawn from **D** pointing towards the centre of the disc.	**1**	When the disc rotates at constant speed, the **only** horizontal force acting on the dust particle is the centripetal force. This acts towards the centre of the circle.
(ii) Centripetal acceleration at position **D** $= \frac{v^2}{r} = \frac{0.59^2}{0.125}$	**1**	If you had calculated the angular speed ω in part **(a)**, you might prefer to calculate the centripetal acceleration using $a = \omega^2\,r$.
$= 2.8$ m s^{-2}	**1**	
(c) *Relevant points*: • A smaller centripetal force is required for particles that are closer to the centre … • because, when the rate of rotation is constant, force $\propto$ radius r ($F = m\omega^2 r$ and ω is constant). • Friction (or electrostatic attraction) is sufficient to hold the dust particles that are closer to the centre but not those further away.	**3**	When a body rotates at a constant rate, the **angular** speed is constant for the whole of the body, but the **linear** speed of a particle in (or on) the body depends on its radius from the axis of rotation. The argument supporting the answer is less clear if you use $F = \frac{mv^2}{r}$ because both v and r change with radius. However, you can link this approach to ω, as follows: $F = \frac{mv^2}{r} = \frac{m(\omega r)^2}{r} = m\omega^2 r$ Hence, for a given mass, $F \propto r$ when ω is constant.

Answers to examination-style questions

Answers	Marks	Examiner's tips
4 (a) Angular speed $\omega = 2\pi f = 2\pi \times \frac{9000}{60}$	1	9000 revolutions per minute is the same as 150 revolutions **per second**. This is the frequency of rotation.
$= 2\pi \times 150$ $= 9.42 \times 10^2$ rad s^{-1}	1	
(b) (i) The centripetal force on the effective mass is applied by the **tension** in the plastic line.	1	The plastic line pulls inwards on the mass all the time it is rotating.
(ii) Centripetal force $F = m\omega^2 r$	1	This calculation needs a little care. In the question the mass is given in g, not kg, and you have to remember to square ω.
$= 0.80 \times 10^{-3} \times (9.42 \times 10^2)^2 \times 0.125$	1	
$= 89$ N	1	
(c) Use of $F\,\Delta t = \Delta(mv)$	1	Part **(c)** is an interesting twist, which revises the work on impulse which is covered in Chapter 1 of *AS Physics A*. The pebble was stationary before being struck by the line, so its change in momentum is (mass) × (velocity acquired).
gives $F \times 0.68 \times 10^{-3} = 1.2 \times 10^{-3} \times 15$	1	
$\therefore$ average force on pebble $F = 26$ N	1	
5 (a) (i) The velocity of the engine changes because the direction of movement changes as it goes round the track.	1	See the more complete answer given (and expected) in Question 1 above. The mark allocation shown alongside each part is a guide to how much you are expected to write. Here it is 2 marks; in Question 1 it is 3 marks.
Acceleration is the rate of change of velocity (or velocity is a vector).	1	
(ii) Arrow drawn towards the centre of the circle on the diagram.	1	A centripetal force is always directed towards the centre of the circular path.
(b) Centripetal force $F = \frac{mv^2}{r} = \frac{0.14 \times 0.17^2}{0.80}$	1	All the necessary data is set out for you to substitute directly into the centripetal force equation. Remember to square v.
$= 5.1 \times 10^{-3}$ N	1	
(c) (i) Centripetal force acts on the **outer** wheel.	1	More insight is required in part **(c)**. The flange of the outer wheel pushes **outwards** against the curved outer rail as the engine attempts to carry on moving in a straight line. The outer rail therefore pushes **inwards** on this same flange, providing the centripetal force.

Answers to examination-style questions

Answers	Marks	Examiner's tips
(ii) *Relevant points include*: • Stress is $\frac{\text{force } F}{\text{area } A}$ • F depends on the mass of the engine, the speed of the engine, and the radius of the track. • A is the area of contact between the wheel and the rail. • A discussion of how changing a physical quantity would affect the stress, for example increasing the mass of the engine would increase the stress, or an increase in the depth of the flange would decrease the stress.	**4**	This revises work on materials covered in Unit 2 of *AS Physics A*. Some of the marks would be available if you were to discuss only the vertical forces on the wheel (due to the weight of the engine), but full marks could only be obtained by discussing the effect of the centripetal force. This is because the question requires you to give an answer 'for the toy engine **going round a curved track**'.
6 (a) (i) Use of tension $F = \frac{mv^2}{r}$ gives $0.35 = \frac{30 \times 10^{-3} \times v^2}{0.45}$ $\therefore$ speed of mass $v = 2.29$ m s^{-1}	**1** **1**	When substituting values, m must be in kg and r in m. Remember to take the square root of v^2 before writing down your answer.
(ii) Period $= \frac{2\pi r}{v} = \frac{2\pi \times 0.45}{2.29}$ $= 1.2$ s	**1** **1**	*Alternatively*: angular speed $\omega = \frac{v}{r} = \frac{2.29}{0.45} = 5.09$ rad s^{-1} period $= \frac{2\pi}{\omega} = \frac{2\pi}{5.09} = 1.2$ s
(b) (i) *Arrows on diagram drawn and labelled as follows*: • **Weight** (or mg), arrow vertically downwards from centre of mass of **M**.	**1**	The mark would not be given for an arrow labelled 'gravity', and the arrow must be drawn carefully, vertically downwards.
• **Tension**, arrow along thread towards centre of circle.	**1**	Labelling the arrow 'centripetal force' would not be acceptable, and the arrow must be **on** the thread, not parallel to it.
• **Air resistance** (or drag), arrow along a tangent to the circle in the opposite direction to the rotation arrow.	**1**	You could easily overlook this force, but it is bound to be present. The mark would not be awarded if you were to label it 'friction'.
(ii) The tension is least when **M** is at the top of the circle and greatest when **M** is at the bottom. At the top: centripetal force = weight + tension $\therefore$ tension $= \frac{mv^2}{r} + mg$ At the bottom: centripetal force = tension − weight $\therefore$ tension $= \frac{mv^2}{r} + mg$	**1** **1** **1**	In this case the centripetal force is the **resultant** force towards the centre of the circle. At the top, both the weight and the tension act in the same direction (vertically downwards). At the bottom, the weight acts downwards whilst the tension acts upwards, so these forces act in opposite directions.

Answers to examination-style questions

Answers	Marks	Examiner's tips
7 (a) Momentum of electron $= mv$ $= 9.1 \times 10^{-31} \times 4.2 \times 10^{7}$ $= 3.8 \times 10^{-23}$ N s (or kg m s^{-1})	1, 1	This first part revises work covered in Chapter 1 of *AS Physics A*.
(b) Magnitude of force on electron $=$ centripetal force $F = \frac{mv^2}{r}$ $= \frac{9.1 \times 10^{-31} \times (4.2 \times 10^{7})^2}{0.045} = 3.6 \times 10^{-14}$ N	1, 1	In examples such as this, where the object moving in a curved path does not move repeatedly around a circle, it is generally best to use the centripetal force equation in the form $F = \frac{mv^2}{r}$, rather than $F = m\,\omega^2 r$.
(c) Arrow drawn from **P** towards the centre **O** of the circular path.	1	This is a further test of the fact that the centripetal force acting on an object is directed to the centre of the circle in which it is moving.
8 (a) *Relevant points*: • A force is needed (or there is an acceleration) towards the centre of the bend. • The movement of the pointer is to the left (or away from the centre). • The right hand spring must stretch to provide this force.	3	The mass inside the accelerometer behaves in much the same way as a passenger in a car going round a bend. **Within the accelerometer**, the mass moves outwards (although it is actually attempting to carry on in a straight line) until the pull of the right hand spring is sufficient to provide the required centripetal force.
(b) (i) Centripetal acceleration $a = \frac{v^2}{r}$ $v = 45$ km h^{-1} $= \frac{45 \times 1000}{3600} = 12.5$ m s^{-1} $\therefore a = \frac{12.5^2}{24} = 6.5$ m s^{-2}	1, 1	The whole car and its contents experience this same acceleration as it travels round the bend. You are required to convert km h^{-1} (which is the usual unit for the speed of a car) into m s^{-1}, as in Question 8.
(ii) Force on mass $= ma = 0.35 \times 6.5$ $= 2.28$ N Movement of pointer $= \frac{2.28}{0.75} \times 27$ $= 82$ mm	1, 1	The force on the mass is the centripetal force, but a has already been calculated in **(b)(i)**. This force will move the pointer $\frac{2.28}{0.75}$ times further than the calibrating force of 0.75 N.

Answers to examination-style questions

Answers	Marks	Examiner's tips
1 (a) (i) With the object on the spring: the mean value of x = 72 mm, e = 70 mm	**1**	
(ii) 1.4%	**1**	Each reading was ±0.5 mm. As the extension was the subtraction of two readings the absolute errors are added to give an absolute error of ±1.0 mm and a percentage error of $\frac{1}{70} \times 100 = 1.4\%$.
(b) (i) 0.551s	**1**	$T_{av} = 11.02$ s
(ii) 0.6%	**1**	The absolute error can be taken as half the range of the values so the error in T_{av} is $\frac{11.11 - 10.97}{2} = 0.07$s.
(c) (i) $mg = ke$ therefore $e = \frac{mg}{k}$	**1**	
(ii) Using the above equation gives $\frac{m}{k} = \frac{e}{g}$	**1**	
Substituting this expression for $\frac{m}{k}$ into the mass–spring time period equation $T = 2\pi\sqrt{\frac{m}{k}}$ gives the required equation	**1**	
(d) Plot either T^2 against e or T against $\sqrt{e}$ to give a straight line.	**4**	One mark each for: • correct labels and units • suitable scales • all points plotted correctly • best-fit line
According to the equation, the line should pass through the origin and the gradient is equal to $\frac{4\pi^2}{g}$ for the T^2 against e line or $\frac{2\pi}{\sqrt{g}}$ for the T against $\sqrt{e}$ line.	**1**	
To determine g, the gradient of the line should be measured.	**1**	
Gradient given the correct unit ($s^2\ m^{-1}$ or $s^2\ mm^{-1}$).	**1**	
A large triangle used correctly to determine the gradient.	**1**	Draw a triangle or use points that cover over half of the line you have drawn. It improves accuracy. Make sure the points used are on your line and not just two of the plotted points.
. . . and used with the appropriate gradient formula above to find g.	**1**	For example, the graph for T^2 against e is shown below. Its gradient = 4.074 $s^2\ m^{-1}$. Hence $g = \frac{4\pi^2}{4.074}$ $= 9.69\ m\ s^{-2}$

Answers to examination-style questions

Answers	Marks	Examiner's tips
(e) The graph should give a best-fit line that passes through the origin (or almost does). Without carrying out detailed error calculations, the percentage errors in the measurement of e and in T suggests an overall percentage error in g of at least 2%, which would give an error in g of $\pm 0.2\ \text{m s}^{-2}$.	1	Alternatively an attempt can be made to find the error in the gradient of the graph by drawing the "worst" possible line as well as the best fit line and comparing the gradients. However with a very good best fit graph just taking readings for the gradient can introduce a small error.
The accepted value of g is within this range.	1	
An improved method of measuring the extension would give a more accurate value of g. Or the extension contributes the largest percentage error and improving this measurement is important.	1	
For example, a convex lens could be used as a magnifying glass to observe the position of the marker pin on the mm scale.	1	
2 (a) (i) Distance d is twice the amplitude.	1	The amplitude A is the maximum displacement from the equilibrium position. In half an oscillation, each prong of the fork oscillates from one rest position, through equilibrium, to the opposite rest position. This is a distance of $2A$.
(ii) *Graph drawn to include*: • At least one cycle of a sinusoidal graph.	1	Start this by drawing axes: displacement/mm (upwards) and time/ms (across the page). 'Sinusoidal' means 'like the shape of a sine wave' (a cosine wave, a −sine wave or a −cosine wave would be satisfactory). The period $T = \frac{1}{f} = \frac{1}{512} = 1.95 \times 10^{-3}$ s Marking this time as $\frac{1}{512}$ s on the axis would not be acceptable.
• Amplitude correctly shown on displacement axis at ±0.85 mm	1	
• Period correctly shown on time axis at 1.95 (or 2.0) m s	1	
(b) (i) Maximum speed of tip of a prong $V_{max} = 2\pi fa = 2\pi \times 512 \times 0.85 \times 10^{-3}$	1	The maximum speed occurs at the centre of the oscillation, where displacement $x = 0$. In general, $v = \pm 2\pi f\sqrt{A^2 - x^2}$; but when $x = 0$ this reduces to $v_{max} = 2\pi fA$.
$= 2.73\ \text{m s}^{-1}$	1	
(ii) Maximum acceleration of tip of a prong $A_{max} = (2\pi f)^2 a$ $= (2\pi \times 512)^2 \times 0.85 \times 10^{-3}$	1	The maximum acceleration occurs at the extremity of an oscillation, where displacement $x = \pm A$. Simple harmonic motion always satisfies the equation $a = -(2\pi f)^2 x$, so a_{max} is found by substituting $x = A$ into this equation.
$= 8.8 \times 10^3\ \text{m s}^{-2}$	1	

Answers to examination-style questions

Answers	Marks	Examiner's tips
3 (a) Period $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{0.800}{9.81}}$ $= 1.79$ s	1 1	Part **(a)** is an easy starter, where the answer is found by direct substitution into the equation for the period of a simple pendulum.
(b) When bob falls, E_P lost $= E_K$ gained $\therefore \frac{1}{2}mv^2 = mg\Delta h$ gives $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 20 \times 10^{-3}}$ $\therefore v_{max} = 0.626$ m s^{-1} but $v_{max} = 2\pi fA$ $\therefore A = \frac{v_{max}}{2\pi f} = \frac{v_{max}T}{2\pi}$ $= \frac{0.626 \times 1.79}{2\pi} = 0.178$ m	1 1 1 1	Part **(b)** is more demanding. You have to realise that you can find the maximum speed (at the lowest point) from conservation of energy. The amplitude can then be found from the maximum speed by applying $v_{max} = 2\pi fA$, together with $f = \frac{1}{T}$. Alternatively, you could find the approximate value of A by applying Pythagoras: $A^2 = 800^2 - 780^2$ gives $A = 178$ mm. $v_{max} = 2\pi fA$ would then give v_{max}.
(c) At lowest point of swing, centripetal force on mass is $(F - mg)$ where F is the tension in the string. $\therefore F - mg = \frac{m{v_{max}}^2}{r}$ Tension $F = m\left(g + \frac{{v_{max}}^2}{r}\right)$ $= 25 \times 10^{-3}\left(9.81 + \frac{0.626^2}{0.800}\right)$ $= 0.257$ N	1 1	Part **(c)** involves some of the work covered in Chapter 2 on circular motion. The mass swings in a circular arc whose radius r is equal to the length of the pendulum. At the lowest point of the circle, the speed is v_{max} and the centripetal force is the **resultant** force towards the centre of the circle. This is (tension – weight of mass).
4 (a) (i) Angular speed of turntable $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.2} = 2.86$ rad s^{-1} Horizontal force on sphere $= m\omega^2 r = 0.050 \times 2.86^2 \times 0.13$ $= 0.053$ N	1 1	This question links simple harmonic motion with a circular model that can generate some of its features. It begins with some useful revision of Chapter 2.
(ii) Towards the centre **of the turntable**.	1	'Towards the centre' would not be sufficient, because it would raise the question '*which* centre?'.
(b) (i) Using $T = 2\pi\sqrt{\frac{l}{g}}$ and rearranging gives $l = \frac{T^2 g}{4\pi^2} = \frac{2.2^2 \times 9.81}{4\pi^2} = 1.20$ m or 1.2 m	1	Direct use of the pendulum equation leads to a straightforward mark. Compare this with Question 3**(a)**, where you had to identify the data with a simple pendulum before you could give an answer.
(ii) Maximum acceleration of bob: $a_{max} = (2\pi f)^2 A = \left(\frac{2\pi}{2.2}\right)^2 \times 0.13$ $= 1.06$ m s^{-2} or 1.1 m s^{-2}	1 1	In this calculation, you have to relate the frequency and period as an intermediate step, by using $f = \frac{1}{T}$.

Answers	Marks	Examiner's tips
(c) *Acceleration–time graph to show*: • $a = 0$ when $t = 0$ • a curve of the correct –sine form, with correct period.	**2**	In shm, acceleration $a \propto -x$, making this graph the mirror image in the time axis of the shape of the displacement graph.
Kinetic energy–time graph to show: • E_K is a maximum when $t = 0$, and E_K is always positive • two cycles for every single cycle of the displacement–time graph, and the correct shape of curve (technically, this is a **(cos)**^{2} graph)	**2**	At $t = 0$, the displacement is zero and so the bob is at its maximum speed; E_K is therefore a maximum at the start. E_K is zero every time the displacement is a maximum (when $t = \frac{T}{4}, \frac{3T}{4}$, etc.), making two cycles for every single cycle of the displacement graph.
5 (a) (i) The minus sign indicates that the acceleration is in the opposite direction to the displacement	**1**	In your answer you must show that you know what the symbols a and × mean. 'a is in the opposite direction to x' would not be worth this mark. Alternatively, you could say that the minus sign means that the restoring force (or acceleration) always acts towards the equilibrium position.
(ii) v–t graph drawn as a sine curve, with $v = 0$ at $t = 0$ and the same period as the a–t graph. (**Phase**: comparing the two sinusoidal graphs, the a–t graph leads the v–t graph by 270° (or $\frac{3\pi}{2}$ rad)	**1**	At $\boldsymbol{t} = \mathbf{0}$, a is at its maximum **positive** value; this shows that × is then at its maximum **negative** value, so $v = 0$. Starting at a point of maximum negative displacement, v then increases positively from zero whilst the motion is towards the equilibrium point. The graph must therefore be a sine curve.
(b) (i) Period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{24}{60}} = 3.97$ s Frequency $f = \frac{1}{T} = \frac{1}{3.97} = 0.252$ Hz $v_{max} = 2\pi f A = 2\pi \times 0.252 \times 0.035$ $= 5.54 \times 10^{-2}$ m s^{-1} Maximum $E_K = \frac{1}{2} m\, v_{max}{}^2$ $= \frac{1}{2} \times 24 \times (5.54 \times 10^{-2})^2$ $= 3.7 \times 10^{-2}$ J (i.e. about 40 mJ)	**1** **1** **1** **1** **1**	This calculation requires you to think through a strategy about the steps to be taken. Many similar calculations are set in a structured way, so that you know the steps to take and the order in which to take them. The vertical displacement of the mass from its equilibrium position (0.035m) is the amplitude A of the shm. Alternatively maximum E_K = maximum $E_P = \frac{1}{2} k x^2 = \frac{1}{2} \times 60 \times 0.035^2 = 37$ mJ
(ii) Graph of E_K against t drawn to show: • Maxima of E_K at $t = 0, 2, 4$ s and minima (zero) at $t = 1, 3$ s • Progressive reduction in size of maxima • $E_K = 40$ mJ at $t = 0$ and $E_K = 10$ mJ at $t = 4.0$ s	**3**	Note that the 'end of a complete cycle' is the period, which is 4.0 s, and choose scales to allow the graph to include 40 mJ and 4.0 s. The maximum $E_K \propto v_{max}{}^2 \propto A^2$, so halving A will reduce the maximum E_K to $\frac{1}{4}$ of its original value. At $t = 1$s and 3s, $v = 0$ and E_K is therefore zero.

Answers to examination-style questions

Answers	Marks	Examiner's tips
6 **(a)** Forced vibrations/oscillations	**1**	**Resonance** occurs only when the forcing vibrations have exactly the same frequency as a natural frequency of the system that is being vibrated. It is doubtful whether the Millennium Bridge was actually in resonance, but it was certainly subject to forced vibrations of large amplitude.
(b) *Relevant points include*: • A structure has a natural frequency (or frequencies) of vibration • Resonance • This occurs when the frequency of a driving force is equal to a natural frequency of the structure • Large amplitude vibrations are then produced (or there is a large transfer of energy to the structure) • This could damage the structure (or cause a bridge to fail)	**any 4**	The condition is that of resonance, which occurs when the frequency of the applied driving oscillator exactly matches a natural frequency of the driven oscillator. It is well known that simple suspension bridges (as used by an advancing army) are subject to this effect, and that troops are instructed to break step when crossing them. The Millennium Bridge, as originally built, presented a particular problem. The vibrating bridge deck fed oscillations back to the pedestrians on it, almost forcing them to walk in step, thereby enhancing its own vibrations.
(c) *Possible measures* to reduce the effect: • install dampers (shock absorbers) • stiffen (or reinforce) the structure • any other acceptable step e.g. redesign to change natural frequency, increase the mass of the bridge, restrict numbers of pedestrians	**any 2**	The Millennium Bridge problem was solved by installing very large shock absorbers, similar in design to those used on motor vehicles. Changing any physical feature of a structure would change its natural frequency, but a bridge over a river should not be shortened!
7 **(a)** **(i)** Period = 1.8 s Frequency $f = \frac{1}{T} = \frac{1}{1.8} = 0.556$ Hz or 0.56 Hz	**1** **1**	If the time axis is read carefully, it is clear that one cycle takes 1.8 s, two cycles 3.6 s and three cycles 5.4 s.
(ii) Amplitude = 0.076 m (± 0.002 m)	**1**	Some variation has to be allowed in this answer, because the vertical scale is not finely calibrated.
(iii) Damping does not alter the frequency … but it does reduce the amplitude	**1** **1**	It is important to realise that the period of vibration (and therefore the frequency) is unaffected by damping. Damping usually removes a fixed proportion of the existing energy in each cycle, and so the amplitude is progressively reduced.

Answers	Marks	Examiner's tips
(b) *Graph drawn to show*: • Maximum +ve displacement at $t = 0$, 1.8, 3.6 s etc and max –ve displacement at $t = 0.9$, 2.7 s, etc. or vice versa **and** zero displacement at $t = 0.45$, 1.35, 2.25, 3.15 s, etc. • Sinusoidal shape, with constant amplitude and period	**2**	This tests your understanding of **phase**. At $t = 0$, the displacement is zero on the original graph. On the new graph (for a 90° phase difference) the displacement at $t = 0$ must be either $+A$ or $-A$. Your graph should be either a cosine curve or a –(cosine) curve. The new graph may be 90° ahead or behind the original graph.
(c) (i) Maximum acceleration of bob $= (2\pi f)^2 A = (2\pi \times 0.556)^2 \times 0.076$ $= 0.93$ m s^{-2}	**1** **1**	The values substituted for f and A are those found from the graph in part **(a)** above.
(ii) Maximum speed of bob $v_{max} = 2\pi f A$ $= 2\pi \times 0.556 \times 0.076 = 0.266$ m s^{-1} Total energy of shm = maximum E_K $= \frac{1}{2} m v_{max}^2$ $= \frac{1}{2} \times 8.0 \times 10^{-3} \times 0.266^2$ $= 2.8 \times 10^{-4}$ J	**1** **1** **1**	In this part you have to realise that the 'total energy of the oscillations' must be equal to the kinetic energy of the bob when its potential energy can be taken to be zero – at the centre of each swing, when it travels fastest. The first step is therefore to find this maximum speed.
8 (a) (i) Use of $m g = k \Delta L$ gives spring constant k $= \frac{mg}{\Delta L} = \frac{0.25 \times 9.81}{40 \times 10^{-3}} = 61.3$ N m^{-1} or 61 N m^{-1}	**1** **1**	This is another question on the mass–spring system that requires you to be familiar with Hooke's law from *AS Physics A* Unit 2.
(ii) Period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.69}{61.3}} = 0.667$ s or 0.67 s Frequency $f = \frac{1}{T} = \frac{1}{0.667} = 1.50$ Hz	**1** **1**	It is essential to read this part of the question carefully. In **(ii)**, a mass of 0.44 kg has been added to the original 0.25 kg, making the total mass supported by the spring 0.69 kg.

Answers	Marks	Examiner's tips
(b) *Relevant points include*: **(i)** (*at 0.2 Hz*) • Forced vibrations at a frequency of 0.2 Hz are produced • The amplitude is similar to the driver's (≈30mm) (or less than at resonance) • The displacements of the masses are almost in phase with the displacements of the support rod **(ii)** (*at 1.5 Hz*) • Resonance is produced (or vibrations at a frequency of 1.5 Hz) • The amplitude is very large (>30 mm) • The displacements of the masses have a phase lag of 90° on the displacements of the support rod • The motion may appear violent **(iii)** (*at 10 Hz*) • Forced vibrations at a frequency of 10 Hz are produced • The amplitude is small (« 30 mm) • The displacements of the masses have a phase lag of almost 180° on the displacements of the support rod	**any 6**	Part **(b)** is still about the system described in part **(a)(ii)**, which has a natural frequency of 1.5 Hz. Resonance will therefore occur if the driving frequency matches this. Any successful answer must refer to **amplitude**, **frequency**, and **phase** in all of the parts, because the question demands this. Resonant systems always vibrate with a phase lag of 90° on the driver. At 0.2 Hz, the driving frequency is much less than 1.5 Hz; the driven system then practically follows the driving vibrations, with a small phase lag. At 10 Hz the driving frequency is much greater than 1.5 Hz, and so the forced vibrations produced are of small amplitude, almost in antiphase with the driver.